Polish J. of Environ. Stud. Vol. 19, No. 4 (2010), 797-804

Original Research

Prediction Models for Minimum and Maximum Dry Density of Non-Cohesive Soils

Maria J. Sulewska*

Faculty of Civil and Environmental Engineering, Białystok Technical University,
Wiejska 45E, 15-351 Białystok, Poland

Received: 2 July 2009
Accepted: 10 February 2010

Abstract

Density index (or relative density) is the measure of compaction of non-cohesive soils naturally compacted as a result of geological processes. It also is used for compaction control of non-cohesive soils built in hydraulic embankments. Hydraulic engineering embankments and hydro-melioration systems should be characterized by high load capacity, tightness, and durability, and low water-permeability and low deformability. These features are obtained as a result of compaction of earth layers. Direct examinations of all parameters required for calculation of density index of each earth layer (dry density of soil in embankment and minimum and maximum dry density of soil) is time consuming.

Dependencies between the dry densities (minimum and maximum) and graining of non-cohesive soils were developed in order to simplify the test procedure of density index. Statistical linear regression and artificial neural networks were applied.

Keywords: compaction evaluation of non-cohesive soils, density index, minimum and maximum dry density, artificial neural networks

Introduction

Examinations of density index (I_D) of built-in soil are conducted during construction, repair work, and periodic control of technical and safety conditions of embankments and hydraulic engineering structures. Basing on the I_D of compaction, compaction quality of non-cohesive soils in subsoil also is determined. It also is used for control of compaction of non-cohesive soils built in hydraulic embankments. Water economics is related to numerous hydraulic structures and hydro-melioration systems. These are mostly earthen structures and different types of embankments such as dams, flood banks, dykes, road embankments, bridge heads, etc. Hydraulic engineering embankments must have high load capacity, leak tightness and durability, and low water permeability and deformability.

Embankments gain required features during their construction as a result of the application of proper soil material and technology of earthen works. The basic technological treatment is building in soil in successive layers with compaction of each layer and control of compaction quality. Examinations of compaction of built-in soil are also conducted during repair works and control of technical and safety conditions of embankments and hydraulic structures.

Control of quality of embankment compaction should lead to determination of proper compaction parameters of every layer built in earth structure, respectively, to a type and class of hydraulic structure [1] and soil type. Requirements and test methods for reception of embankments of hydro-melioration systems are regulated by Polish standard PN-B-12095 [2]. Requirements related to hydraulic engineering embankments are given in technical conditions of construction and reception published by the Ministry of Environmental Protection, Natural Resources

*e-mail: sulewska@pb.edu.pl

Table 1. Required values of I_s or I_D [2].

Soil type	Content of grains >2mm [%]	Required compaction		
		Body of earth dams	Body of new banks	
			I, II class of importance	III, IV class of importance
Cohesive soils	0-10	$I_s \geq 0.95$	$I_s \geq 0.95$	$I_s \geq 0.92$
	10-50	$I_s \geq 0.92$	$I_s \geq 0.92$	
Non-cohesive soils	Fine sands	$I_D \geq 0.75$	$I_D \geq 0.70$	$I_D \geq 0.55$
	Medium sands	$I_D \geq 0.70$		
	Coarse sands and coarse-grained soils	$I_D \geq 0.65$	$I_D \geq 0.65$	

and Forestry in 1994 [3]. Criteria and procedure of evaluation of technical conditions and safety of earthen anti-flood structures are given in the guidelines [4, 5].

The measures of compaction quality of material in hydraulic structure are the following geotechnical parameters: degree of compaction (I_s) in the case of cohesive soils, or density index (I_D) for non-cohesive soils. Control of soil compaction in an embankment consists of the comparison between the obtained value of compaction measure and the required value (I_s or I_D). The value of measurement of soil compaction should be at least equal to the required value given in Table 1.

This study refers to the possibility of simplifying the test procedure of I_D for non-cohesive soils by eliminating laboratory examinations of minimum dry density (ρ_{dmin}) and maximum dry density (ρ_{dmax}) of soil, which are necessary for I_D calculation. These parameters can be determined based on the developed dependencies. Statistical and neural methods were applied for modelling the parameters ρ_{dmin} and ρ_{dmax} based on soil graining.

Experimental Procedures

The I_D (called also relative density D_R in literature [6]) is still extensively used in geotechnical engineering as an index of the mechanical properties of coarse-grained soils. It is evaluated either by field tests or by laboratory tests. Field penetration tests are various kinds of sounding techniques such as the standard penetration test SPT [7, 8], dynamic penetration tests [8, 9] and cone penetration tests [10-12]. Geotechnical engineers need to estimate the *in situ* I_D using empirical correlations between I_D and penetration test results. This indirect way of evaluating I_D adds further uncertainties to those already faced when determining the I_D in the laboratory [13].

This study refers to the possibility of simplifying laboratory test procedures of I_D of non-cohesive soils during control of quality compaction of each layer built in an embankment.

I_D quantity indicates position of the field void ratio (e) between the maximum void ratio (e_{max}) and minimum void ratio (e_{min}) according to the equation:

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{\rho_{d\,max}}{\rho_d} \cdot \frac{\rho_d - \rho_{d\,min}}{\rho_{d\,max} - \rho_{d\,min}} \tag{1}$$

The two extreme void ratios e_{max} and e_{min} are therefore not unique, but they depend on the methods used for their determination [6, 14].

Dry density (ρ_d) of soil in embankment is calculated according to the equation:

$$\rho_d = \frac{100\rho}{100 + w} \tag{2}$$

Volume density (ρ) and soil water content (w) should be tested in situ and more samples should be taken for further laboratory examinations of compaction parameters in order to determine I_D in the tested embankment layer. Compaction parameters such as ρ_{dmin} and ρ_{dmax} are determined in laboratory [15]. Laboratory examination of ρ_{dmin} and ρ_{dmax} consists in the examination of dry specimen at the most loose and the most dense grain arrangement (Fig. 1). The most loose arrangement of grains is obtained by pouring sand through a funnel into a cylinder (height h=12.54 cm and diameter D=7.10 cm). The most dense arrangement of grains is obtained by compacting the soil in the cylinder using a vibrating fork.

Laboratory examinations are the most time consuming stage of the controlling process. For this reason the trials to eliminate or simplify these examinations are undertaken. Examinations of different types of non-cohesive soil are described in literature. On this ground it can be observed that the values of ρ_{dmin} and ρ_{dmax} of soil depend on different factors, especially on soil type and graining and the method of compaction [16, 17].

The aim of this study was developing the models for prediction of the densities (ρ_{dmin} and ρ_{dmax}) on the basis of graining parameters of non-cohesive soils:

a) index of graining uniformity (C_U)

$$C_U = \frac{d_{60}}{d_{10}} \tag{3}$$

Table 2. Geotechnical parameters of tested soils.

Type of soil	Silty sands (P_π)	Fine sands (P_d)	Medium sands (P_s)	Coarse sands (P_r)	Sand and gravel mixes (P_o)	Gravels ($\dot{Z}$)	Total
Number of patterns	21	47	24	13	11	5	121
ρ_{dmin} [g/cm^3]	1.253-1.569	1.247-1.578	1.320-1.632	1.458-1.746	1.612-1.881	1.591-1.773	1.247-1.881
ρ_{dmax} [g/cm^3]	1.643-1.849	1.604-1.903	1.701-1.869	1.751-2.019	1.850-2.112	1.982-2.124	1.604-2.124
C_U	1.27-4.75	1.25-2.53	1.26-4.57	3.32-7.52	2.90-12.50	7.38-10.26	1.25-12.50
d_{10} [mm]	0.019-0.125	0.080-0.170	0.140-0.360	0.170-0.250	0.165-0.540	0.390-0.500	0.019-0.500
d_{20} [mm]	0.040-0.170	0.100-0.215	0.180-0.430	0.195-0.420	0.250-0.830	0.740-1.000	0.040-1.00
d_{30} [mm]	0.040-0.200	0.120-0.240	0.200-0.450	0.280-0.500	0.360-1.200	1.150-2.200	0.040-2.20
d_{40} [mm]	0.060-0.215	0.120-0.250	0.220-0.470	0.340-0.930	0.500-1.600	2.000-2.850	0.060-2.85
d_{50} [mm]	0.074-0.220	0.120-0.260	0.250-0.800	0.420-1.300	0.630-2.350	2.450-3.500	0.070-3.50
d_{60} [mm]	0.084-0.250	0.125-0.300	0.260-0.660	0.630-1.730	0.830-3.750	2.950-4.500	0.084-4.500
d_{70} [mm]	0.093-0.300	0.128-0.320	0.260-0.850	0.860-1.700	1.150-6.000	3.650-5.900	0.093-6.000
d_{80} [mm]	0.110-0.395	0.135-0.450	0.270-1.200	1.150-2.500	1.700-10.000	4.900-9.000	0.110-10.000
d_{90} [mm]	0.140-0.400	0.150-0.850	0.320-1.850	1.500-5.000	2.350-25.000	8.000-17.500	0.140-25.000

b) grain diameters d_x (in mm), below which remains x % of soil weight, where x=10, 20, …, 90, at Δx=10%.

The range of laboratory test results is presented in Table 2. The analyzed set of data contained 121 cases.

Results

Examinations of the analyzed geotechnical parameters were conducted for several groups of non-cohesive soils (silty sands, fine sands, medium sands, coarse sands, sand and gravel mixes, and gravels) according to PN-88/B-04481 [15]. Laboratory examinations were made on specimens of natural soils from Białystok and on soils purposely prepared in order to obtain diversified graining.

Models of Linear Regression

Statistical and neural analyses were made using STATISTICA software [18, 19]. Correlations between variables were analyzed based on matrix of coefficients of linear regression (Table 3). It was found that there were statistically significant linear correlations between densities (ρ_{dmin} or ρ_{dmax}) and graining parameters (C_U, d_{10}, d_{20}, d_{30}, d_{40}, d_{50}, d_{60}, d_{70}, d_{80}, d_{90}). But the most influential grain diameters

a) b)

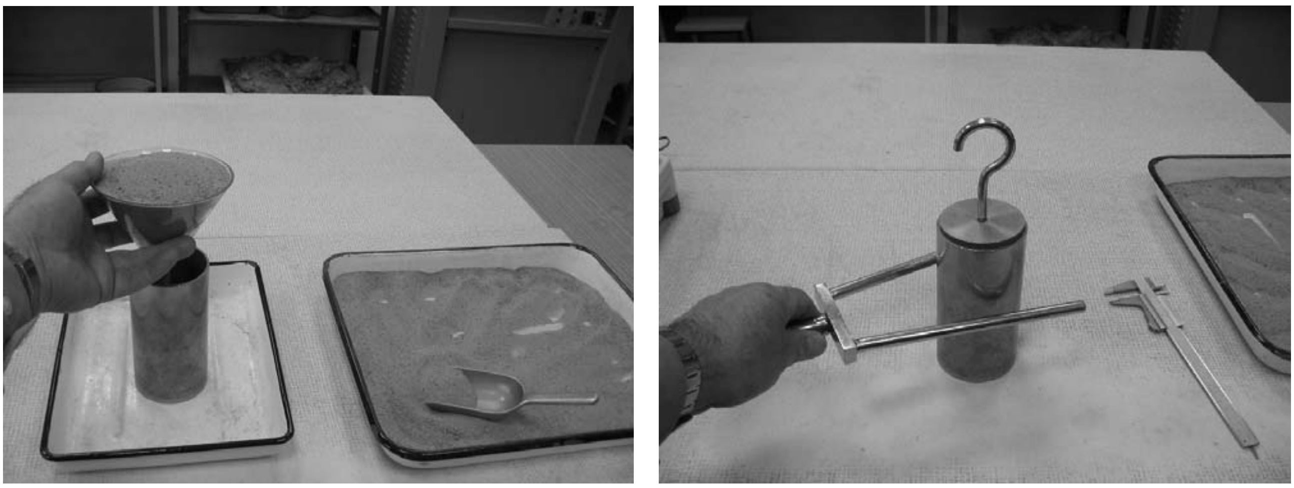

Fig. 1. Test procedure: a) ρ_{dmin}, b) ρ_{dmax} according to PN-88/B-04481 [15].

Table 3. Matrix of coefficients of linear correlation.

Variable	ρ_{dmin}	ρ_{dmax}	C_U	d_{10}	d_{20}	d_{30}	d_{40}	d_{50}	d_{60}	d_{70}	d_{80}	d_{90}
ρ_{dmin}	1.00											
ρ_{dmax}	0.93	1.00										
C_U	**0.58**	**0.71**	1.00									
d_{10}	**0.70**	**0.69**	0.55	1.00								
d_{20}	**0.66**	**0.72**	0.66	0.94	1.00							
d_{30}	**0.63**	**0.71**	0.73	0.86	0.97	1.00						
d_{40}	**0.62**	**0.72**	0.78	0.84	0.95	0.99	1.00					
d_{50}	**0.64**	**0.74**	0.84	0.83	0.93	0.96	0.99	1.00				
d_{60}	**0.66**	**0.76**	0.88	0.81	0.89	0.93	0.97	0.99	1.00			
d_{70}	**0.68**	**0.77**	0.90	0.78	0.84	0.88	0.92	0.95	0.98	1.00		
d_{80}	**0.67**	**0.75**	0.86	0.74	0.77	0.80	0.85	0.89	0.93	0.98	1.00	
d_{90}	**0.57**	**0.62**	0.62	0.62	0.65	0.68	0.69	0.72	0.75	0.82	0.88	1.00

could not be distinguished – all diameters (d_x) influence compaction parameters to a similar degree. The analyzed values of the correlation coefficients are given in bold. The models of simple linear correlation have linear correlation coefficients in the range r = 0.58 to r = 0.77.

The graphs of linear dependencies between ρ_{dmin} or ρ_{dmax} and index C_U are given in Fig. 2.

Then the linear models of multiple regression of parameters ρ_{dmin} and ρ_{dmax} were developed. Graining parameters (C_U, d_x) served as explanatory variables in these models. Only C_U and diameter d_{10} were statistically significant variables. After taking into consideration only statistically significant variables the following multiple regression models, which are slightly better than models of simple linear regression, were obtained (r values are given in Table 3). The models of multiple regression are expressed by the following equations:

$$\rho_{dmin}=1.276+0.031C_U+1.238d_{10}\pm0.089, \quad r = 0.775 \tag{4}$$

$$\rho_{dmax}=1.608+0.038C_U+0.637d_{10}\pm0.071, \quad r = 0.816 \tag{5}$$

a) b)

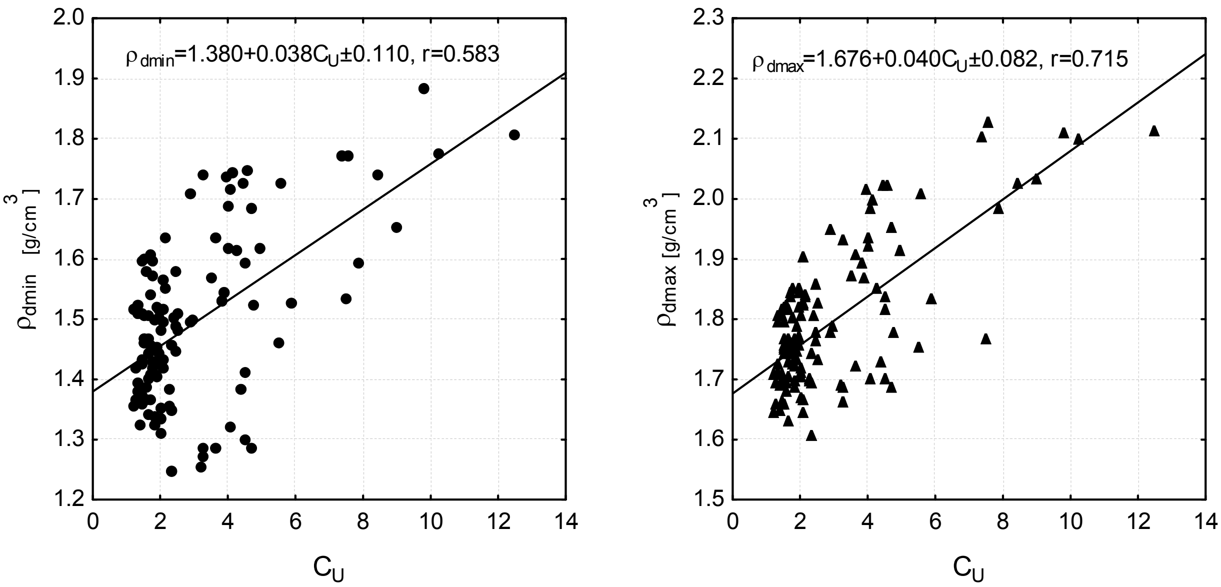

Fig. 2. Dependencies: a) ρ_{dmin}=f(C_U), b) ρ_{dmax}=f(C_U).

Table 4. Error measurements of analyzed ANNs.

Output architecture number of epochs	Inputs	RMS			MAE_i			r_i		
		L	*V*	*T*	*L*	*V*	*T*	*L*	*V*	*T*
ρ_{dmin} 10-4-1 QN 43	C_U, d_{10}, d_{20}, d_{30}, d_{40}, d_{50}, d_{60}, d_{70}, d_{80}, d_{90}	0.109	0.106	0.121	0.050	0.052	0.065	0.852	0.806	0.904
ρ_{dmax} 10-4-1 QN 134		0.108	0.145	0.131	0.040	0.055	0.047	0.859	0.842	0.838
ρ_{dmin}		0.108	0.119	0.135	0.046	0.048	0.057	0.883	0.881	0.878
ρ_{dmax} 10-3-2 QN 970					0.041	0.047	0.055	0.857	0.821	0.868

Artificial Neural Networks

Artificial neural networks (ANN) are the modern information tool that enables modelling of complex and multidimensional phenomena in many fields. This new method of data analysis is used, among other things, for modelling complex issues in environmental protection [20-23], engineering [24, 25], and in water economics [26].

Multilayer feed-forward networks with one hidden layer were applied for solving the analyzed regression problems.

The networks were optimized with regard to the number of input variables, the number of neurons in the hidden layer and the learning method. Selection of the optimal network structure was made empirically. ANN learned on examples, as a model based on learning data. The set of patterns consisted of cases described by the values of input variables and the corresponding output variables. The whole set of cases (*P*) was randomly divided into three subsets: learning subset (*L*) with 61 cases, validating subset (*V*) with 30 cases and testing subset (*T*) with 30 cases. The criterion for stopping the learning process was minimization of validation root mean squared error (RMS).

The aim of the learning process was determining the values of weights of neuron connections of all network layers in such a manner that at the given input vector ($\mathbf{x}^{(p)}$) it was possible to obtain the values of output signals ($\mathbf{y}_i^{(p)}$) with sufficient accuracy equal to the demanded values ($d_i^{(p)}$) (where i is the number of input and p is the number of pattern from *P*). Learning algorithms were of iteration type. All cases from the learning set were given to the input in each iteration step, called epoch. For each case the output value was calculated. This value was compared to the true value. The difference between these two values (i.e. error) was used to adjust the weights in the network in such a manner that the error had the lowest value.

The criterion for stopping the learning process was minimizing RMS validation. The variable metric method turned out to be the most effective learning method (called Quasi Newton QN method in STATISTICA). This method is precisely described in [27-29].

The best networks were selected on the basis of the lowest value of RMS in validating and testing subsets:

$$RMS = \sqrt{\frac{1}{PM}\sum_{p=1}^{P}\sum_{i=1}^{M}\left(d_i^{(p)} - y_i^{(p)}\right)^2} \tag{6}$$

...where $d_i^{(p)}$ is the true output value, $y_i^{(p)}$ is the calculated value corresponding to $d_i^{(p)}$, i is number of input (i = 1, …M), and p is number of pattern from the considered set (p = 1, …, P).

Other network error measurements were also analyzed, such as:

- mean absolute error MAE, i.e. the average value from error modulus $|d_i^{(p)} - y_i^{(p)}|$,
- correlation coefficient (r) between the true and calculated values of explained variable.

In the figures presenting accuracy of prediction there are also marked the areas of relative errors (RE) calculated according to the equation:

$$RE_i^{(p)} = \left|\frac{d_i^{(p)} - y_i^{(p)}}{d_i^{(p)}}\right| 100\% \tag{7}$$

At the beginning the networks with one output variable were analyzed. ANNs with 10 inputs (C_U, d_{10}, d_{20}, d_{30}, d_{40}, d_{50}, d_{60}, d_{70}, d_{80}, d_{90}), one hidden layer (with 4 neurons) and one output ρ_{dmin} or ρ_{dmax} were applied. The errors of both 10-4-1 ANNs are given in Table 4.

ANN with one hidden layer and two outputs ρ_{dmin} and ρ_{dmax} was used for simultaneous modelling of both compaction parameters. The scheme of 10-3-2 ANN is presented in Fig. 3. Error measurements for each output of 10-3-2 ANN are presented in Table 4.

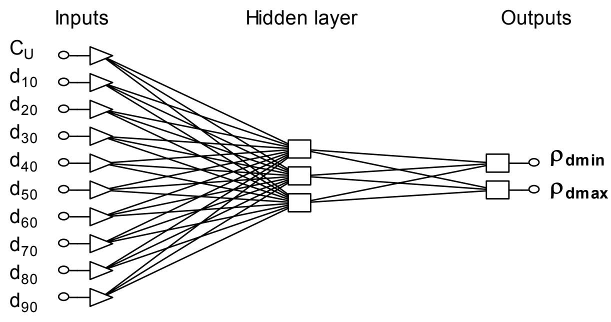

Fig. 3. The ANN scheme of 10-3-2 architecture.

The learning graphs of ANNs are shown in Fig. 4. During ANN training the early stopping algorithm was applied. This means that learning was stopped when the value of error in the validation subset started to grow [19, 27].

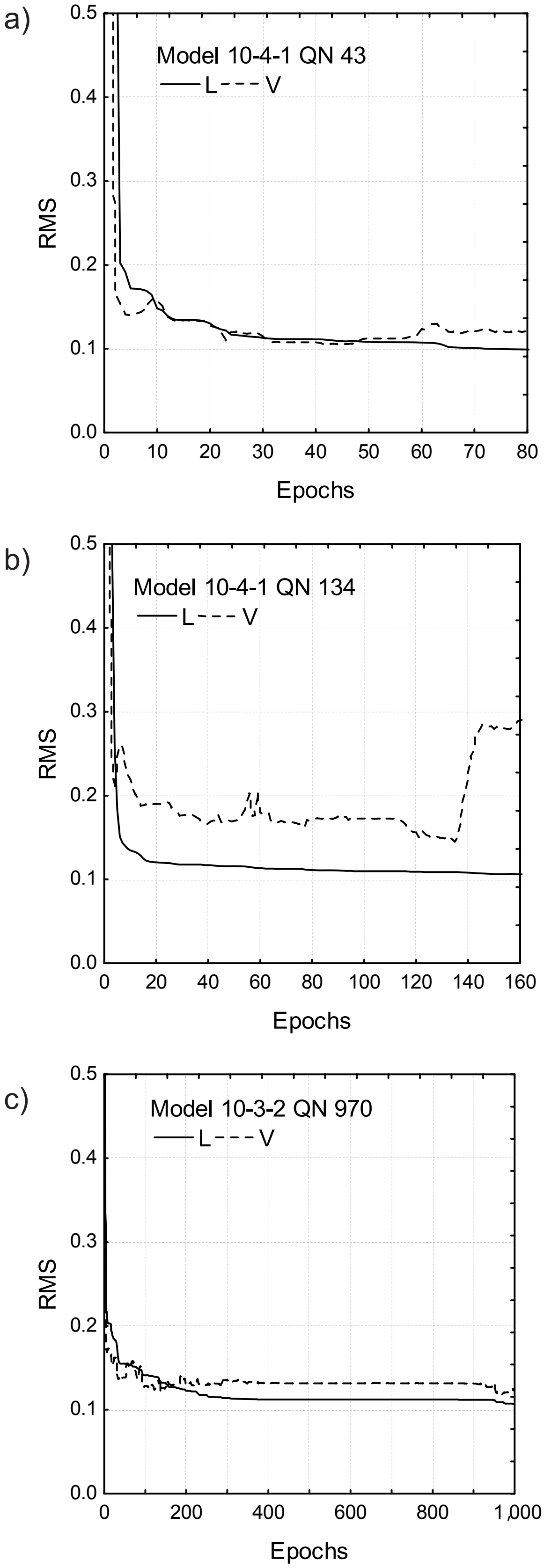

Fig. 4. RMS variation with iteration for training and validation stages.

Table 5. Comparison of error measurements of the analyzed prediction models.

Parameter	ρ_{dmin}			ρ_{dmax}		
Model	Linear (5)	10-4-1	10-3-2	Linear (6)	10-4-1	10-3-2
r	0.775	0.904	0.878	0.816	0.838	0.868
RE [%]	13	10	10	10	8	8

Summarizing the analysis of error measurements of ANNs combined in Table 4, it can be concluded that the quality of ANN with one output is comparable to the quality of ANN modelling 2 parameters simultaneously. The quality of prediction of the compaction dry densities ρ_{dmin} and ρ_{dmax} is satisfactory.

Comparisons between the observed values and the values predicted by both 10-4-1 ANNs and 10-3-2 ANNs (with RE area) constitute an illustration of the above conclusions (Fig. 5). The parameter ρ_{dmin} is predicted by ANN with RE equal to approximately 10% and the parameter ρ_{dmax} – with RE equal to approximately 8%.

For comparison, prediction accuracy of the parameters based on equations (4) and (5) are presented in Fig. 6. Parameter ρ_{dmin} can be predicted by equation (4) with RE equal to approximately 13%, and parameter ρ_{dmax} can be predicted by equation (6) with RE equal to approximately 10%.

The prediction quality of the analyzed linear and neural models can be most easily compared based on the values of correlation coefficients and relative errors (Table 5).

Conclusions

Standard methods of I_D determination are time-consuming and involve several steps and a rapid method of estimation would therefore be useful. The empirically derived equations in this paper allow rapid and inexpensive estimation of the I_D for non-cohesive soils.

The model of multiple correlation allows determining ρ_{dmin} with RE equal to approximately 13% and ρ_{dmax} – with RE equal to approximately 10%. Application of ANN allows improving the prediction quality of the analyzed parameters. Parameter ρ_{dmin} is predicted by 10-4-1 ANN with RE equal to approximately 10%, and parameter ρ_{dmax} is predicted with RE equal to approximately 8%. ANN of 10-3-2 architecture models simultaneously ρ_{dmin} and ρ_{dmax} with accuracy equal to 10% and 8%, respectively.

Acknowledgements

The work presented in this paper was supported by Polish financial resources on science in 2007-10 as a Rector's project.

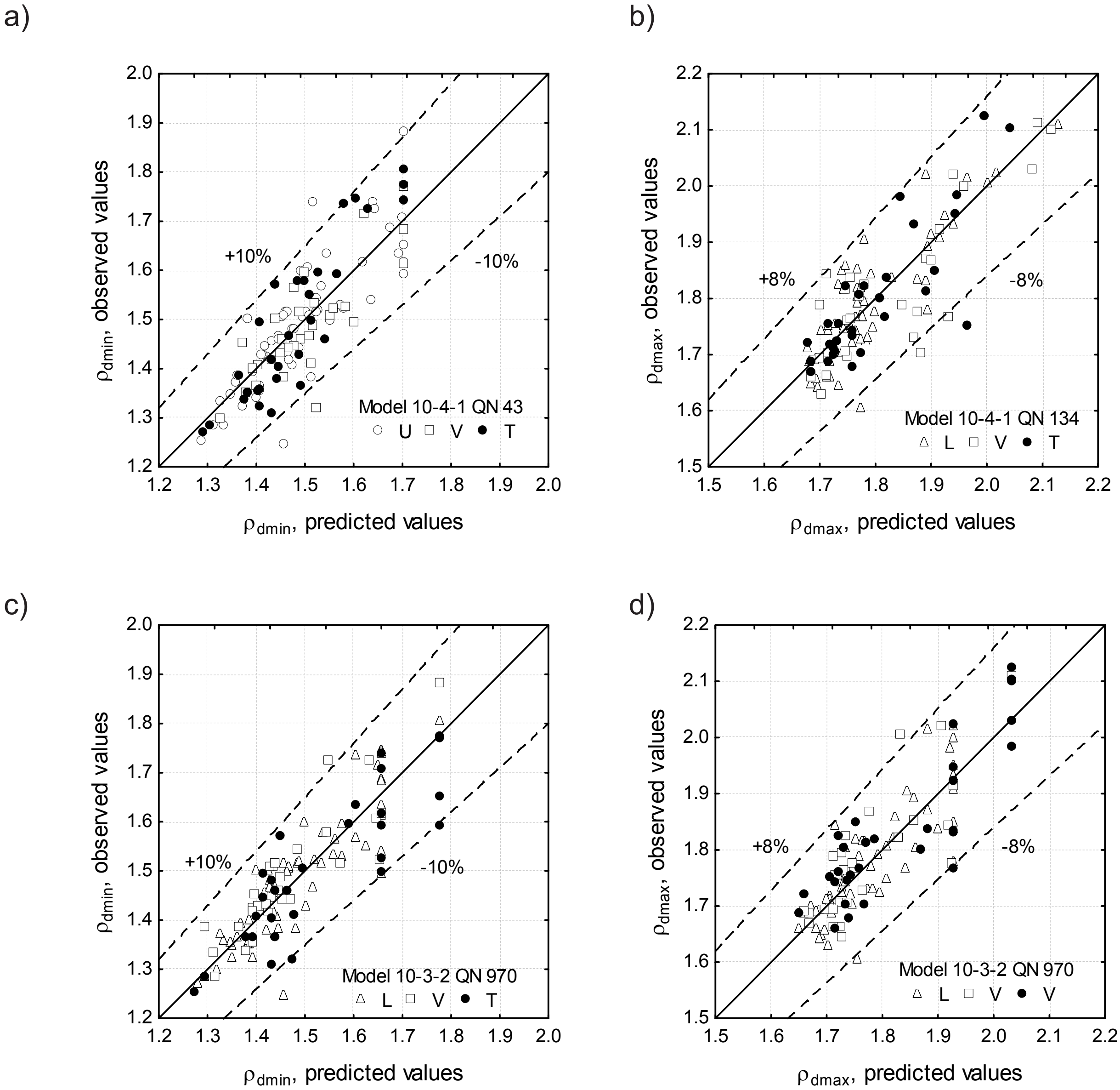

Fig. 5. Comparisons between the observed values and the values predicted by: a) 10-4-1, b) 10-4-1, c) and d) 10-3-2 ANNs.

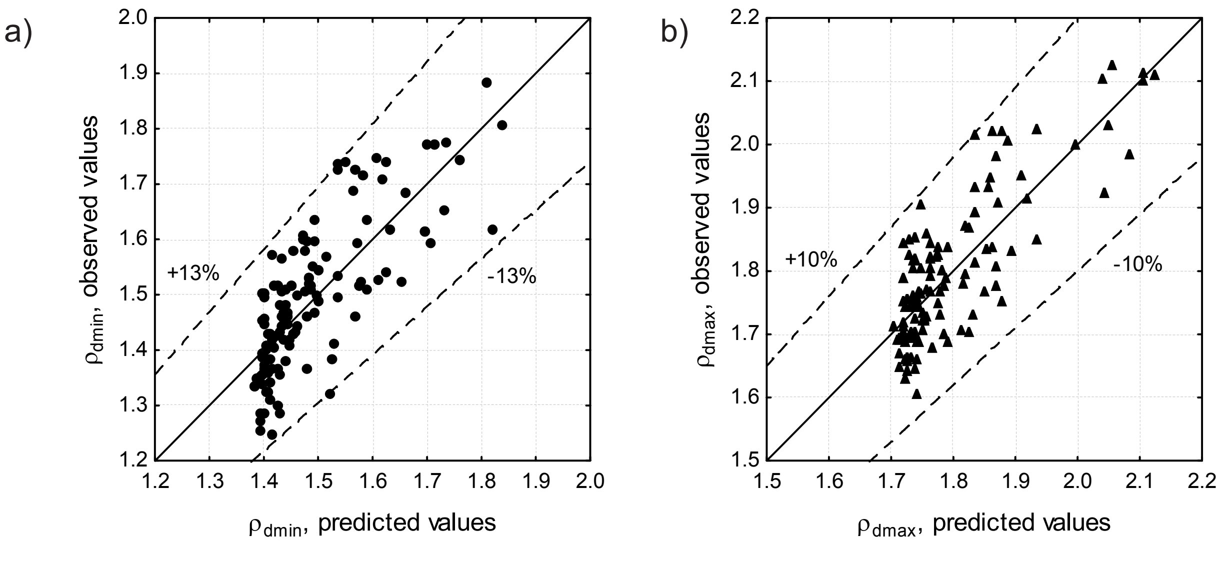

Fig. 6. Comparison between the values of parameters predicted by equations (4) and (5), and the observed values: a) ρ_{dmin}, b) ρ_{dmax}.

References

1. Regulation of Minister of Environmental Protection, Natural Resources and Forestry from 20th April on technical conditions for water economics structures and their location. Act Register, No. **86**, item 579, **2005** [In Polish].
2. Polish standard PN-B-12095 Hydro-melioration systems. Embankments. Requirements and examinations at reception, **1997** [In Polish].
3. Technical conditions of construction and reception. Earthen works. Ministry of Environmental Protection, Natural Resources and Forestry: Warsaw, **1994** [In Polish].
4. BORYS M., MOSIEJ K. Instruction containing criteria and evaluation scale of technical conditions and safety of flood banks, Publisher IMUZ: Falenty, **2006** [In Polish].
5. BORYS M., MOSIEJ K. Regulations for evaluation of technical conditions and safety of flood banks, Publisher IMUZ: Falenty, **2006** [In Polish].
6. LADE P. V., LIGGIO C. D., YAMAMURO J. A. Effects of non-plastic fines on minimum and maximum void ratios of sand. Geotech. Test. J., **21**, (4), 336, **1998**.
7. TSUKAMOTO Y., ISHIHARA K., SAWADA S. Correlation between penetration resistance of Swedish weight sounding test and SPT blow counts in sandy soils. Soils and Foundations, **44**, (3), 13, **2004**.
8. CABRERA M., CARCOLÉ A. Relationship between standard penetration test (SPT) and dynamic penetration super heavy test (DPSH) for natural sand deposits. Proc. of the 14th European Conference on Soil and Geotechnical Engineering Madrid 2007, Millpress: Rotterdam, pp. 1691-1695, **2007**.
9. DIN 4094-3: 2002 Subsoil – Field testing – Part 3: Dynamic probing, **2002** [In German].
10. MŁYNAREK Z. Site investigation and mapping in urban area. Proc. of the 14th European Conference on Soil and Geotechnical Engineering Madrid 2007, Millpress: Rotterdam, pp. 175-202, **2007**.
11. SCHNAID F. Geo-characterisation and properties of natural soils by *in situ* tests. Proc. of the 14th International Conference on Soil Mechanics and Geotechnical Engineering Osaka 2005, Millpress: Rotterdam, pp. 3-45, **2005**.
12. JUANG C. H., LU P. C., CHEN C. J. Predicting geotechnical parameters of sands from CPT measurements using neural networks. Computer-Aided Civil and Infrastructure Engineering, **17**, (1), 31, **2002**.
13. JAMIOLKOWSKI M., LO PRESTI D. C. F., MANASERRO M. Evaluation of relative density and shear strength of sands from CPT and DMT, Monograph of Wydawnictwo SGGW: Warsaw, pp. 107-144, **2006**.
14. CUBRINOVSKI M., ISHIHARA K. Maximum and minimum void ratio characteristics of sands. Soils and Foundations, **42**, (6), 65, **2002**.
15. Polish standard PN-88/B-04481 Building soils. Soil samples testing, **1988** [In Polish].
16. BARTON M. E., CRESSWELL A., BROWN R. Measuring the effect of mixed grading on the maximum dry density of sands. Geotech. Test. J., **24**, (1), 121, **2001**.
17. MUSZYNSKI M. R. Determination of maximum and minimum densities of poorly graded sands using a simplified method. Geotech. Test. J., **29**, (3), 263, **2006**.
18. STANISZ A. Intelligible Course of Statistics Basing on STATISTICA PL on Examples from Medicine, StatSoft Polska: Kraków, **1**, **2001** [In Polish].
19. STATISTICA Neural Networks PL. Guidebook, LULA P., TADEUSIEWICZ R. (Eds), StatSoft Polska: Kraków **2001** [In Polish].
20. MAIER H. R., DANDY G. C. Neural network based modelling of environmental variables: a systematic approach, Mathematical and Computer Modelling. **33**, (6-7), 669, **2001**.
21. PENG G., LESLIE L. M., SHAO Y. Environmental Modeling and Prediction. Springer-Verlag: Berlin, Heidelberg, **2002**.
22. CHAN C. W., HUANG G. H. Artificial intelligence for management and control of pollution minimization and mitigation processes. Engineering Applications of Artificial Intelligence, **16**, (2), 75, **2003**.
23. PARUELO J. M., TOMASEL F. Prediction of functional characteristics of ecosystems: a comparison of artificial neural networks and regression models. Ecol. Model. **98**, (2-3), 173, **1997**.
24. RAFIQ M. Y., BUGMANN G., EASTERBROOK D. J. Neural network design for engineering applications. Computers and Structures, **79**, (17), 1541, **2001**.
25. SHAHIN M. A., JAKSA M. B., MAIER H. R. Artificial neural network applications in geotechnical engineering. Australian Geomechanics, **36**, (1), 49, **2001**.
26. TWARÓG B. Application of neural networks for modelling of using the environment. Gaz, Woda i Technika Sanitarna, **11**, 63, **2006** [In Polish].
27. BISHOP Ch. M. Neural Networks for Pattern Recognition, Oxford University Press Inc.: New York, **1995**.
28. OSOWSKI S. Neural Networks for Information Processing, Oficyna Wydawnicza Politechniki Warszawskiej: Warszawa, **2006** [In Polish].
29. ROJAS R. Neural Network. A Systematic Introduction, Springer-Verlag: Berlin, Heidelberg, **1996**.